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# Off-lattice Eden-C cluster growth model

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**Abstract.** A non-trivial cluster growth model, equivalent to the lattice-free Eden-C model, is proposed. The model is constructed by randomly adding contiguous circles without overlapping. Large-scale computer simulations show the interior density is constant at 0.650, while the boundary is fractal, with a thickness proportional to the 0.396 power of the mean radius.

## 1. Introduction

In 1961 Eden [1] introduced a stochastic growth model which may be used to study the propagation of epidemics, chemical reactions, tumour growths, forest fires and percolation theory. The algorithm is essentially as follows. On a square lattice a cell is labelled as 'infected'. Then any one of the four possible adjacent cells is randomly chosen to be infected. The pair now has six possible growth surfaces. The process continues until a cluster is formed [2, 3]. It is found that the cluster is compact [4], i.e. has a solid core. Three versions of the Eden model have been introduced [5]. In version A, a to-be-infected cell is chosen with same probability from all uninfected cells adjacent to the cluster. In version B, an infection path from all possible paths from infected to adjacent uninfected cells is chosen with the same probability (the original Eden model). In version C, firstly a boundary cell of the cluster is randomly chosen, then an uninfected adjacent cell is randomly chosen to be infected. All three models give similar but somewhat different boundary statistics.

The Eden cluster, however, is anisotropic due to the underlying lattice, i.e. its shape tends to lengthen along the directions of the lattice axes. In comparison to diffusion-limited aggregation (DLA) models [6, 7] the Eden anisotropy is much weaker [8–10], the orientational differences being about 2–2.5% in growth velocity and 9–11% in surface width [11]. However, even small anisotropies wreak havoc in the estimation of the cluster boundary width [8, 12]. Thus most analyses to date simulated Eden growth on a strip [13, 10] where such directional preferences could be more uniform than the Eden cluster.

Even for the strip geometry the numerical simulations for the boundary properties are still very difficult. Convergence to the asymptotic state is slow and large numbers of cells are needed. In order to accelerate the process, a 'noise reduction' method was used on the strip geometry with apparent success [14]. However, the same method applied to the cluster greatly enhances anisotropy, such that the Eden cluster rapidly changes from circular to rhombic shape [15, 16]. Excellent reviews of the current situation were written by Vicsek [17] and Meakin [12].

In order to exclude the troublesome lattice effects, some off-lattice (no lattice) models have been proposed. Meakin [18] found that the result of the off-lattice DLA model differs

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from that of the square lattice. Jullien and Meakin [19] studied the off-lattice ballistic deposition model. The only existing off-lattice *Eden model* was introduced by Botet [20]. The cluster is composed of touching circles instead of squares. The centre of a possible additional circle can only be on connected arcs (may be multiply connected) enclosing the cluster. Define a continuous line connecting all arcs and choose at random a point on this line to be the centre of the new growth. Botet found that the interior of the cluster shows distinct holes which differs from the compact Eden interior. Botet's pioneer off-lattice Eden model is equivalent to the version-A Eden model where all possible sites are chosen with the same probability. Unfortunately the samples are somewhat small, perhaps due to the fact that its numerical implementation is quite expensive.

Since *only* an off-lattice Eden model can correctly predict the properties of isotropic natural growth (although for some crystals the growth can be anisotropic), it is imperative to investigate more deeply such stochastic growth models. In the current study we shall present the other basic growth model, namely the off-lattice version-C Eden cluster model which complements the work of Botet.

## 2. The model and its properties

The algorithm is described as follows.

(i) On a plane a living cell (a circle) is introduced. Only living cells are capable of growth.

(ii) A cell is selected randomly from the existing living ones. The range of directions along which an adjacent touching cell can be grown (attached) without overlapping any existing cells is identified. A random direction is chosen from this admissible range and a new cell is grown.

(iii) If there are no possible growth directions, the cell is labelled dead.

The process is similar to the Eden model version C whereby one chooses a possible filled site then fills a possible adjacent site. However, the present model is lattice-independent. In what follows we shall investigate the statistical properties of this model using large-scale computer simulations.

A typical result, after a growth of 3000 cells, is shown in figure 1. The actual cell membranes of course are not circles but are Voronoi–Dirichlet polygons surrounding their centres. Unlike Botet's version-A model, living cells exist only on the cluster boundary (figure 2). We see that the interior cells are almost evenly distributed but the cluster boundary is uneven.

Using the centre of mass as the origin, one can draw concentric circles of radius $r$ (radius of one cell = 1 unit) and count the number of cells inside. Figure 3 shows a typical result for 200 000 cells. We see that the number of cells is proportional to $r^2$ in the interior, i.e. the density $\rho$ (ratio of cell area to domain area) is constant. We used 100 samples of $2 \times 10^5$ cells and established the value

$$E[\rho] = 0.6500 \tag{1}$$

with a standard deviation of 0.000 77. Notice that the interior density of the Eden squares is 1, the density of the most closely packed circles (each with six contacts) is $\pi/2\sqrt{3} = 0.9069$, and that of a random dense packing of circles (each with about three contacts) is 0.772 [21].

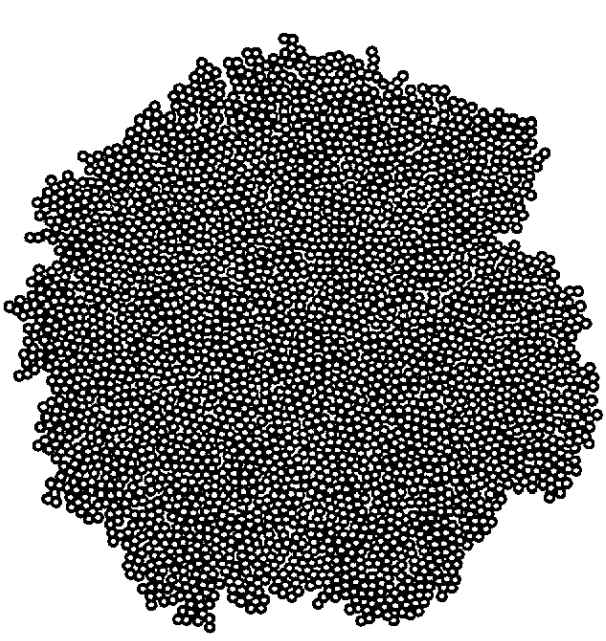

**Figure 1.** A cluster of 3000 cells.

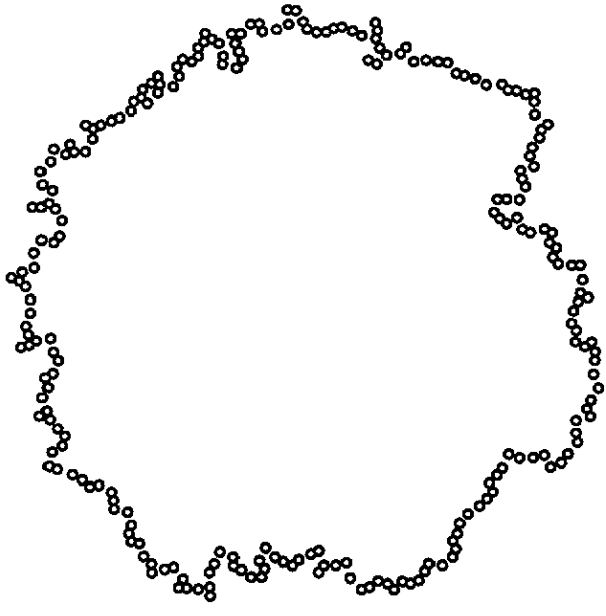

**Figure 2.** Living cells of figure 1.

Next we looked at the living cells in the boundary. Figure 4 shows the number of living cells $L$ is proportional to the mean radius $R$ of the boundary. We find

$$L = 3.580R. \tag{2}$$

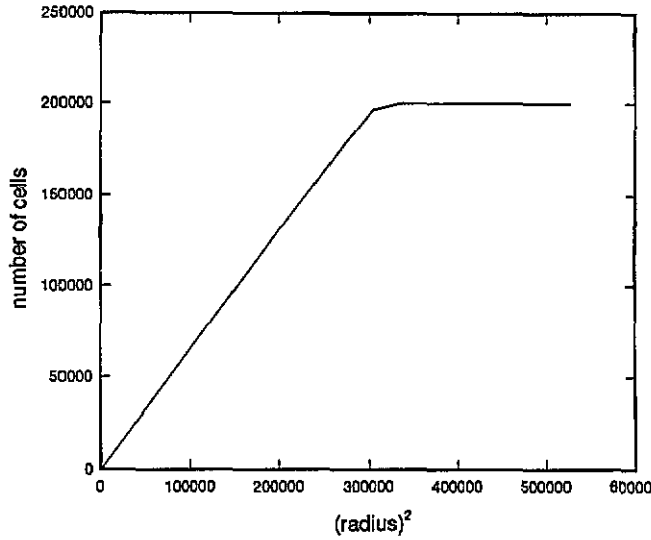


**Figure 3.** Number of cells enclosed by a circle of radius $r$, centred at the centre of mass (a typical sample).

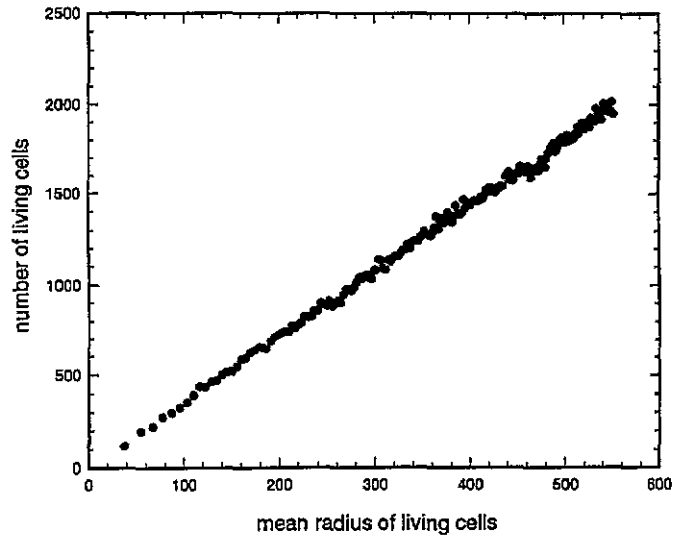


**Figure 4.** Number of living cells versus its mean radius (a typical sample).

The slope has a standard deviation of 0.0153. The 'thickness' of the boundary can be quantified by the standard deviation $\sigma$ of the cell radial distance. Figure 5 shows a typical result using $2 \times 10^5$ cells. We find, with 100 samples,

$$\sigma = 0.8813R^{0.396}. \tag{3}$$

The value of 0.396 is close to the universal value of $\frac{1}{3}$ predicted by Kardar, Parisi and Zhang [22] using a continuum theory based on a nonlinear Langevin equation.

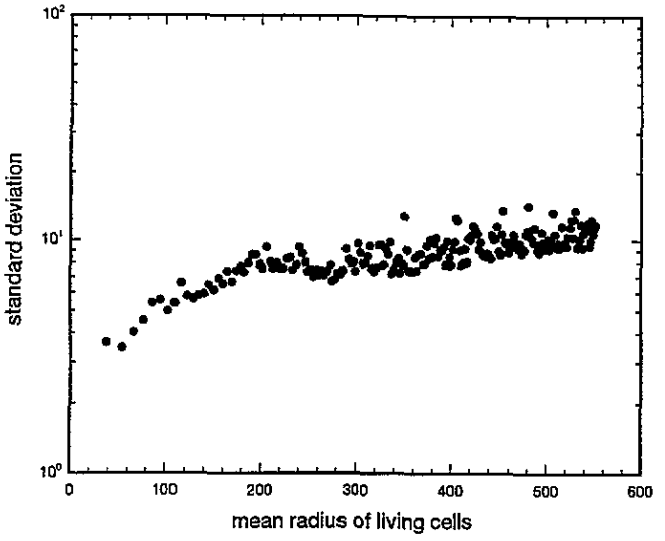


**Figure 5.** The thickness of the boundary represented by the standard deviation from its mean radius (a typical sample).

## 3. Discussion

Since the use of a lattice in the algorithm greatly affects the resulting predictions of stochastic growth, the importance of the off-lattice models cannot be over-emphasized, even though such models are far more expensive in their implementation.

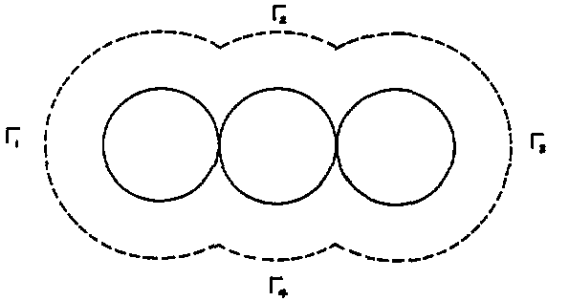


**Figure 6.** Three touching cells on a straight line. Dashed curves show possible locations of the centre of a new growth.

Now let us compare the current off-lattice Eden-C model with Botet's off-lattice Eden-A model [20]. Consider three colinear contiguous circles of radius 1 (figure 6). The centre of the next circle added to the cluster would be somewhere on the arcs (dashed lines of radius 2) indicated by $\Gamma_1$, $\Gamma_2$, $\Gamma_3$, $\Gamma_4$. In Botet's version A, the probability of an added circle whose centre lies on the arc $\Gamma_2$ is

$$\frac{\text{arc length of } \Gamma_2}{\text{arc length of } \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4} = \frac{1}{10}. \tag{4}$$

In the current version C, the probability of choosing the middle circle is $\frac{1}{3}$, then the probability of choosing $\Gamma_2$ from the admissible $\Gamma_2$ and $\Gamma_4$ is $\frac{1}{2}$. Thus the total probability is $\frac{1}{6}$, a value much larger than that of version A. The growth process of the two versions would be different. There are no equivalent version-B off-lattice Eden models.

A major difference of the results is that version A shows numerous holes, capable of accepting new growth in the interior, while version C shows no such holes. As indicated in the example above, version C seems to favour the growth of interior cells, thus the interior becomes more compact. Another possible explanation is that version A is known to have slow equilibriation and strong finite-size corrections, at least for the latticed Eden model [23] and perhaps for the off-lattice models as well. Whether versions A and C approach the same asymptotic state awaits further study. If the actual growth has no interior holes, perhaps the version-C model is more appropriate than version A.

Both off-lattice Eden models are based on substrate depletion principles which govern most biological growth. In fact these are the simplest (and most basic) non-trivial isotropic growth models.

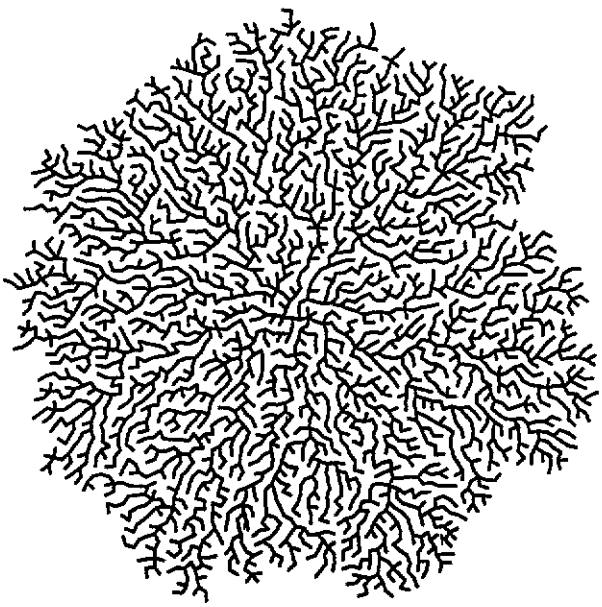

**Figure 7.** A growth network from figure 1.

Recently there has been some interest in using the Eden model to generate growth networks [24–26]. By connecting the centres of continuous cells the present model can also generate dense branching networks. An example is shown in figure 7, which is the skeleton of figure 1. The statistics of the growth tips of the tree are exactly the same as that of the Eden-C cluster.

## Acknowledgment

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